Junior Maths Mastery Challenge Sample

Paper C

Section A

Questions 1 to 5 carry 3 marks each.

1. Find the missing term in the pattern below.

1, 2, 3, 6, 11, 20, 37, ______, 125, …

[Patterns and sequences]

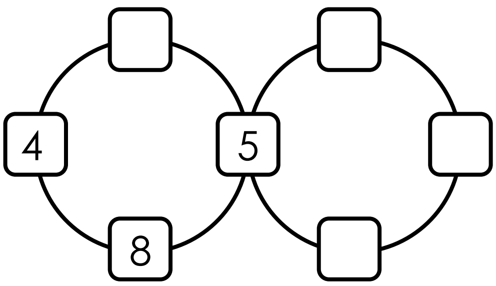
1 + 2 + 3 = 6
2 + 3 + 6 = 11
3 + 6 + 11 = 20
6 + 11 + 20 = 37
From 6 onwards, each term is the sum of its 3 preceding terms.

11 + 20 + 37 = 68
20 + 37 + 68 = 125
The missing term is 68.

(A) 63 (B) 57 (C) 74
(D) 81 (E) None of the above

2. In the magic circle below, each box is to be filled in with a whole number from 3 to 10 such that the sum of the numbers along each circle is equal. What is the greatest possible sum of the numbers along each circle?
(Each number can only be used once.)

[Logical reasoning]



The remaining numbers we can fill are 3, 6, 7, 9 and 10.
4 + 8 + 3 = 15
We cannot form the sum 15 using three of the remaining numbers.
4 + 8 + 6 = 18
We cannot form the sum 18 using three of the remaining numbers.
4 + 8 + 7 = 19
3 + 6 + 10 = 19
The sum of the numbers along each circle is 19 + 5 = 24.
4 + 8 + 9 = 21
We cannot form the sum 21 using three of the remaining numbers.
4 + 8 + 10 = 22
6 + 7 + 9 = 22
The sum of the numbers along each circle is 22 + 5 = 27.

(A) 24 (B) 25 (C) 26
(D) 27 (E) None of the above

3. A company rented some boats for 52 employees.
The boats could seat either 6 or 8 of them.
A 6-seater boat cost $12 to rent. An 8-seater boat cost $15 to rent. What was the minimum amount the company had to pay to rent the boats? [Problem solving]

It costs less if the company can rent as many 8-seater boats as possible.

Number of people (6-seater)	Number of people (8-seater)	Divisible by 8
6	46	×
12	40	✓

$12 \div 6 = 2$
$40 \div 8 = 5$
$2 \times \$12 = \$24$
$5 \times \$15 = \$75$
$\$24 + \$75 = \$99$
The company had to pay a minimum of $99 to rent the boats.

(A) $102 (B) $105 (C) $108
(D) $123 (E) None of the above

4. There are 6 blue, 8 yellow, 10 red and 12 green blocks in a box. Without looking into the box, Ken removes 1 block at a time from the box. What is the minimum number of blocks he has to remove so that, for certain, he will obtain 2 blocks of different colours? [Number Theory]

We need to consider the worst case scenario where Ken picks a block of a different colour in the last pick.

This scenario will be him removing 12 green blocks and either 1 blue, yellow or red as his last pick.

$12 + 1 = 13$
He has to remove a minimum of 13 blocks to be certain that there will be 2 blocks of different colours.

(A) 7 (B) 9 (C) 11
(D) 13 (E) 31

5. A bakery sells 6 types of muffins — blueberry, chocolate, matcha, red velvet, strawberry and vanilla. Eric wants to buy 2 different types of muffins. How many different combinations can he have? [Combinatorics]

If Eric chooses blueberry muffins, he can choose another type from chocolate, matcha, red velvet, strawberry and vanilla. There are 5 different combinations.

If he chooses chocolate muffins, he can choose another type from matcha, red velvet, strawberry and vanilla. There are 4 different combinations.
Note that we do not need to consider blueberry muffins for the second type since we have accounted for the combination above.

If he chooses matcha muffins, he can choose another type from red velvet, strawberry and vanilla. There are 3 different combinations.

If we continue this way of choosing the combinations, we will have the following:
$5 + 4 + 3 + 2 + 1 = 15$
He can have 15 different combinations.

(A) 15 (B) 16 (C) 18
(D) 20 (E) 25

Questions 6 to 10 carry 4 marks each.

6. In the cryptarithm below, each letter represents a different digit.

	A	B	C
+	A	B	C
D	E	B	A

[Cryptarithm]

Find the greatest possible number ABC represents.

C + C = A or C + C = 1A
So, A must be an even number.

Since ABC + ABC gives a 4-digit number, D = 1.
The possible digit A can be is 6 or 8.

	A	B	C
+	A	B	C
1	E	B	A

Let A = 8 since we are looking for the greatest possible number ABC represents.
The possible digit B can be is 0 or 9.
B can be 9 only if C + C = 18 but this means that C = 9. This is not possible since B and C are different digits.
So, B = 0.

	8	0	C
+	8	0	C
1	E	0	8

C + C = 8. So, C = 4.
So, E = 6.

	8	0	4
+	8	0	4
1	6	0	8

The greatest possible number ABC represents is 804.

(A) 603 (B) 698 (C) 803

(D) 804 (E) None of the above

7. What is the first number from the left in the 10th row of the following pattern?

Row 1				1			
Row 2			2	3	4		
Row 3		5	6	7	8	9	
Row 4	10	11	12	13	14	15	16

⋮

[Patterns and sequences]

Let's list down the first number from the left of each row. We will get the following:
1, 2, 5, 10, 17, 26, 37, 50, 65, 82

The first number from the left in the 10th row is 82.

(A) 84 (B) 86 (C) 88

(D) 90 (E) None of the above

8. Joe adds up consecutive numbers from 1 onwards, 1 + 2 + 3 + 4 + 5 + …. He writes the sum on a piece of paper as he adds the numbers mentally. When he reaches the sum 260, he realises that he has forgotten to add one number. What is the number?

[Arithmetic]

Let's add up numbers 1 to 10.
1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = 55

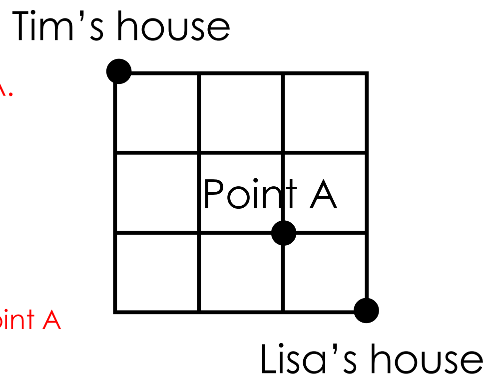
11 + 12 + 13 + 14 + 15 + 16 + 17 + 18 + 19 + 20 = 155
(This is the same as adding 10 to each number in the row above.)

Adding up numbers from 1 to 20, the sum is 55 + 155 = 210
210 + 21 = 231
231 + 22 = 253
253 + 23 = 276
The only possible way is forgetting to add one number when he has added up numbers 1 to 23.

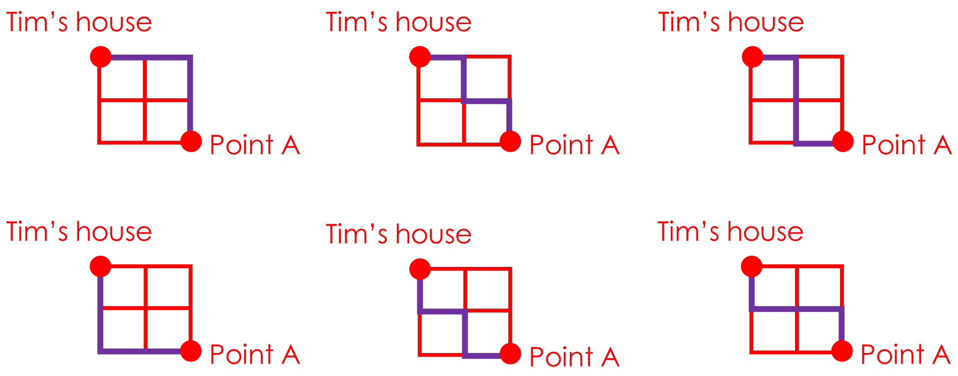
276 – 260 = 16
He has forgotten to add the number 16.

(A) 16 (B) 18 (C) 20
(D) 22 (E) 24

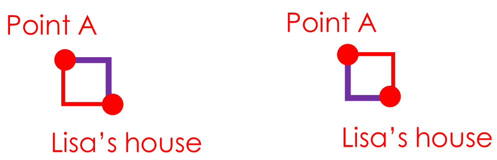
9. The lines in the diagram show the paths from Tim's house to Lisa's house. Tim wants to take the shortest path from his house to Lisa's house, passing through Point A. How many different ways are there? [Combinatorics]



Count the number of ways to walk from Tim's house to Point A.
There are 6 different ways.



Count the number of ways to walk from Point A to Lisa's house.
There are 2 different ways.



$6 \times 2 = 12$
There are 12 different ways.

(A) 12 (B) 14 (C) 16
(D) 18 (E) 20

10. There were 36 pupils in a classroom. The teacher gave each of them a role, Truth-teller or Liar.

The pupils were given 15 minutes to walk around and shook hands with any other pupil only once. A pupil could choose not to shake hands with anyone. When two pupils shook hands, they would reveal their role only to each other.

After 15 minutes, the teacher asked the pupils,
'How many Truth-tellers did you shake hands with?'

Each pupil answered according to their roles.
A Truth-teller had to give the correct answer and the Liar had to give the incorrect answer. Each pupil gave a different answer, 0, 1, 2, 3, …, 33, 34 and 35.

How many pupils were Liar(s)? [Logical reasoning]

The pupil who said he shook hands with 35 Truth-tellers must be a Liar.
Because if he shook hands with 35 Truth-tellers, he must have shaken hands with the pupil who said he had shaken hands with 0 Truth-tellers. This is a contradiction.
So, the pupil who said he shook hands with 35 Truth-tellers was a Liar.

The pupil who said he shook hands with 34 Truth-tellers could not have shaken hands with the pupil who was a Liar (shown above). This means he had shaken hands with the pupil who said he had shaken hands with 0 Truth-tellers. This is a contradiction.
So, this pupil was also a Liar.

Similarly, pupils who said they shook hands with 1, 2, 3, 4, …, 35 Truth-tellers were Liars.
How about the pupil who said he shook hands with 0 Truth-tellers?
If he was a Truth-teller, then he was right that he shook hands with 0 Truth-tellers or he just did not shake hands with any pupil.

If he was a Liar, then he did shake hands with some Truth-tellers but this is not possible as we have shown that the other pupils were Liars.

Therefore, this pupil is the only 1 Truth-teller. 35 pupils were Liars.

(A) 0 (B) 1 (C) 18

(D) 35 (E) 36

Section B

Questions 11 and 12 carry 6 marks each.

11. Jane has a digital clock. The clock is set to show time in 24-hour format. From 0:00 to 12:00, how many times does the digit 4 appear?
(You need not include the digits in the seconds.
At 1:44, the digit 4 appears 2 times.
At 1:45, the digit 4 appears 1 time.) [Combinatorics]

Count the number of times the digit 4 appears from 0:00 to 0:59.
We can count the number of times the digit 4 appears in the whole numbers from 1 to 59.
From 1 to 39, the digit 4 appears 4 times, i.e. 4, 14, 24 and 34.
From 40 to 49, the digit 4 appears 11 times, i.e. 40, 41, 42, 43, 44, …, 49.
From 50 to 59, the digit 4 appears 1 time, i.e. 54.

From 0:00 to 0:59, the digit 4 appears 4 + 11 + 1 = 16 times.
From 1:00 to 1:59, the digit 4 appears 16 times.
From 2:00 to 2:59, the digit 4 appears 16 times.
Similarly, when the hour digit is 3 to 11, the digit 4 appears 16 times each in the minutes.
$12 \times 16 = 192$
The digit 4 appears 192 times in the minutes from 0:00 to 12:00.

Now, let's count the number of times the digit 4 appears when the hour digit is 4.
From 4:00 to 4:59, the digit 4 appears 60 times as the hour digit.

192 + 60 = 252
The digit 4 appears 252 times from 0:00 to 12:00.

12. A leap year has 366 days, in which there are 29 days in February. In a certain leap year, there are 5 Fridays in the month of February. On which day of the week is the last day of that year?

[Problem solving]

To have 5 Fridays in the month of February, the only possible way is for the first Friday to fall on 1 Feb, 8 Feb, 15 Feb, 22 Feb and 29 Feb.

If we count backwards, 31 Jan is a Thursday as well as 24 Jan, 17 Jan, 10 Jan and 3 Jan. So, 1 Jan is a Tuesday.

There are 366 days in that year.
$365 \div 7 = 52$ R 1
31 Dec, the last day of that year, is 52 weeks and 1 day after 1 Jan.

52 weeks after 1 Jan is a Tuesday and 1 day after will be Wednesday.

The last day of that year is a Wednesday.