Junior Maths Mastery Challenge Sample

Paper E

Section A

Questions 1 to 5 carry 3 marks each.

1. Find the value of the following.

$$1 - 2 + 3 - 4 + 5 - 6 + \ldots + 2023 - 2024 + 2025$$ [Arithmetic]

We can rearrange the numbers as shown below.

$1 + \underbrace{3-2}_{1} + \underbrace{5-4}_{1} + \underbrace{7-6}_{1} + \ldots + \underbrace{2023-2022}_{1} + \underbrace{2025-2024}_{1}$

$= 1 + \underbrace{1 + 1 + 1 + \ldots + 1 + 1}_{1012}$

$= 1013$

$2025 - 2 + 1 = 2024$
$2024 \div 2 = 1012$
There are 1012 pairs of terms.

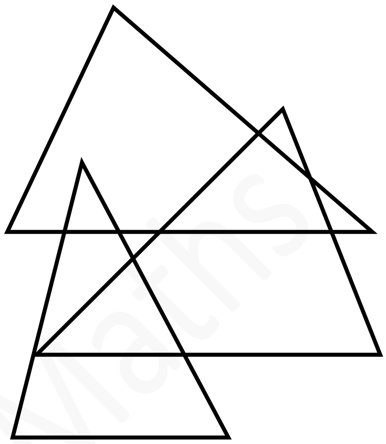
(A) 1012 (B) 1013 (C) 1014
(D) 1015 (E) None of the above

2. Leon's watch slowed at a constant rate of 10 minutes every hour. At 10 a.m., he adjusted his watch to the correct time. What was the correct time when his watch showed 2 p.m. on the same day? [Problem solving]

50 min on the watch $\rightarrow$ 60 min in actual time
1 min on the watch $\rightarrow$ $\frac{6}{5}$ min in actual time
From 10 a.m. to 2 p.m., the duration is 4 h = 240 min.
240 min on the watch $\rightarrow$ $\frac{6}{5}$ min $\times$ 240
= 288 min in actual time
= 4 h 48 min in actual time
The correct time was 2.48 p.m.

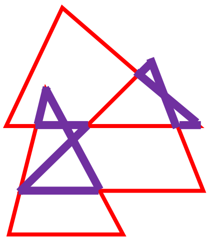
(A) 1.20 p.m. (B) 2:20 p.m. (C) 2.40 p.m.
(D) 2.48 p.m. (E) 2.50 p.m.

3. The figure is made up of three overlapping triangles.
How many triangles are there in the figure? [Spatial visualisation]

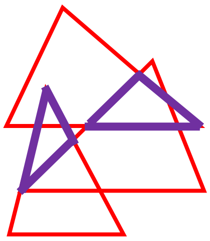


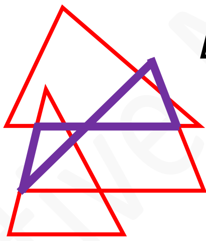
The figure itself is made up of 3 overlapping triangles.

Count the smallest possible triangles formed by the overlapping.
There are 5 such triangles.

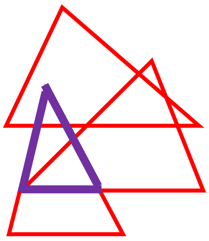


Count the triangles formed by the overlapping and also contain one triangle.
There are 4 such triangles.





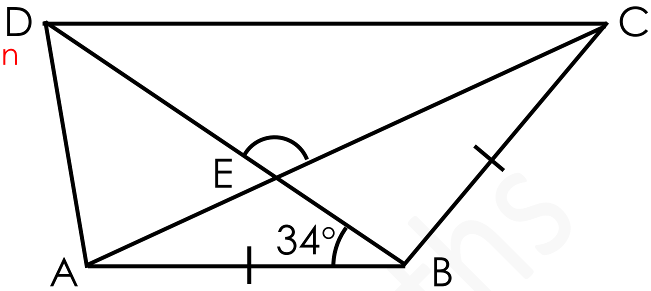
Count the triangles formed by the overlapping and also contain more than one triangle.
There is 1 such triangle.



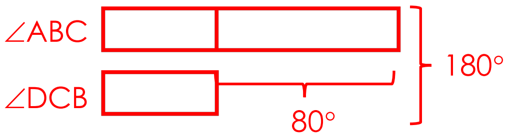
3 + 5 + 4 + 1 = 13
There are 13 triangles in the figure.

(A) 12 (B) 13 (C) 14
(D) 15 (E) None of the above

4. In the figure, AB is parallel to DC and AB = BC.
The difference in angle size between ∠ABC and ∠DCB is 80°.
Find ∠DEC. [Geometry]



∠ABC + ∠DCB = 180° (The sum of angles between two parallel lines is 180°.)



2 units = 180° – 80°
= 100°
1 unit = 100° ÷ 2
= 50°
∠ABC = 50° + 80°
= 130°

∠CAB = (180° – 130°) ÷ 2 (The sum of angles in a triangle is 180°. / The base angles of an isosceles triangle are equal.)
= 25°
∠AEB = 180° – 25° – 34° (The sum of angles in a triangle is 180°.)
= 121°
∠DEC = ∠AEB (Vertically opposite angles are equal.)
= 121°

(A) 116° (B) 121° (C) 126°

(D) 131° (E) None of the above

5. Tim and Paul had some marbles in the ratio 3 : 4.
After Paul gave some marbles to Tim, the ratio of the number of Paul's marbles to that of Tim's marbles became 1 : 2. What was the smallest possible number of marbles Paul gave Tim?

[Problem solving / Use Before-After concept – Unchanged total]

Before

Tim	:	Paul	Total
3		4	7
9	:	12	21

×3 ×3 ×3

After

Tim	:	Paul	Total
2		1	3
14	:	7	21

×7 ×7 ×7

12 – 7 = 5
Paul gave 5 units of marbles to Tim.

The smallest possible number of marbles Paul gave Tim is if 1 unit = 1 marble.

The smallest possible number of marbles was 5.

(A) 3 (B) 5 (C) 7

(D) 10 (E) None of the above

Questions 6 to 10 carry 4 marks each.

6. Square tiles are used to form some figures. The figures follow the pattern below.

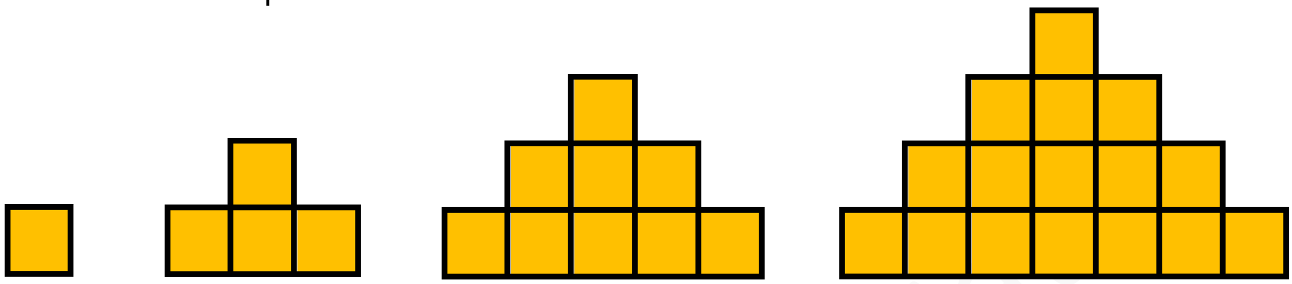


Figure 1 Figure 2 Figure 3 Figure 4

How many square tiles are used to form Figure 99?

[Patterns and sequences]

Number of tiles
Figure 1: $1 = 1 \times 1$
Figure 2: $1 + 3 = 4 = 2 \times 2$
Figure 3: $1 + 3 + 5 = 9 = 3 \times 3$
...
Figure 99: $99 \times 99 = 9801$

9801 square tiles are used to form Figure 99.

(A) 9604 (B) 9702 (C) 9801
(D) 9900 (E) None of the above

7. 654 and 920 are examples of 3-digit numbers with their digits in descending order. How many such 3-digit numbers are there? [Combinatorics]

The smallest possible value of such number is 210.

If the digit in the hundreds place is 3, the digit in the tens place can be 2 or 1.
321, 320
310
2 + 1 = 3
There are 3 such numbers.

If the digit in the hundreds place is 4, the digit in the tens place can be 3, 2 or 1.
432, 431, 430
421, 420
410
3 + 2 + 1 = 6
There are 6 such numbers.

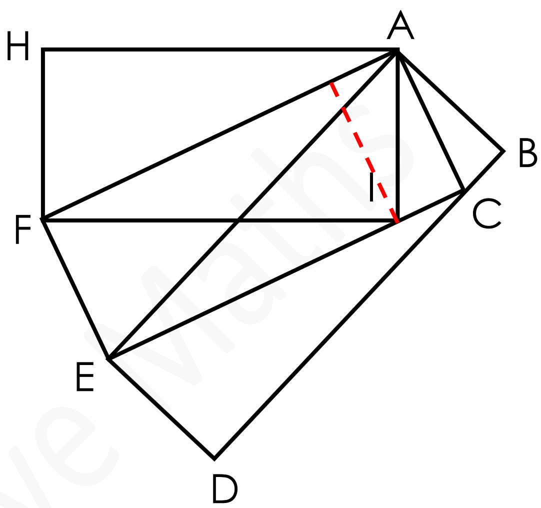
If the digit in the hundreds place is 5, the digit in the tens place can be 4, 3, 2 or 1.
543, 542, 541, 540
532, 531, 530
521, 520
510
4 + 3 + 2 + 1 = 10
There are 10 such numbers.

Following this pattern, we will have the following:
1 + 3 + 6 + 10 + 15 + 21 + 28 + 36 = 120
There are 120 such 3-digit numbers.

(A) 45 (B) 90 (C) 120
(D) 180 (E) None of the above

8. The figure is made up of rectangles ABDE, ACEF and AIFH. The length of HA is 30 centimetres and the length of HF is 15 centimetres. What is the area of Rectangle ABDE?

[Mensuration / Simplify the problem]



Observe that the area of Triangle AIF is half of the area of Rectangle AIFH. The area of the triangle is also half of the area of Rectangle ACEF.
So, the area of rectangles AIFH and ACEF are equal.

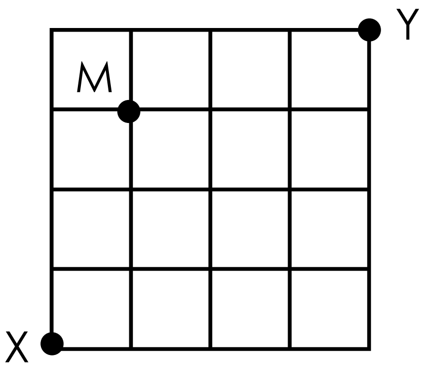
Similarly, the area of Triangle ACE is half of the area of Rectangle ACEF and also half of the area of Rectangle ABDE.
So, the area of rectangles ACEF and ABDE are equal.

The area of the three rectangles are equal.

$30 \times 15 = 450$
The area of Rectangle ABDE is 450 cm^2.

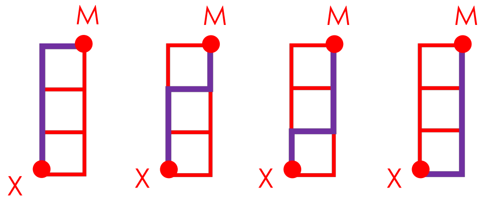
(A) 270 cm^2 (B) 225 cm^2 (C) 180 cm^2
(D) 150 cm^2 (E) None of the above

9. The lines in the diagram show the paths from Point X to Point Y. Ken wants to take the shortest path from Point X to Point Y, passing through Point M. How many different ways are there for him to walk from Point X to Point Y?



[Combinatorics]

Let's walk from Point X to Point M. There are 4 different ways.



Observe that the size of the grid from Point M to Point Y is the same as that from Point X to Point M.

So, the number of different ways to walk from Point M to Point Y is also 4.

$4 \times 4 = 16$

There are 16 different ways for him to walk from Point X to Point Y.

(A) 8 (B) 10 (C) 12

(D) 14 (E) None of the above

10. Ali, Ben, Cheryl and Don are playing a game 'Truth-teller and Liars'. The Truth-teller always speaks the truth and the Liars always lie.

Each of them draws a card and is playing the role of either a Truth-teller or a Liar.

Each of them made the following statement.

Ali: Exactly one of us is a Liar.
Ben: Exactly two of us are Liars.
Cheryl: Exactly three of us are Liars.
Don: All of us are Liars.

[Logical reasoning]

At least one of them is a Truth-teller. Which of the following statements is **true**?

At least one of them is a Truth-teller.
Don must be a Liar because we are told that at least one of them is a Truth-teller.

If Ali is the Truth-teller and we let Don be the one Liar Ali is referring to, then Ben and Cheryl are Truth-tellers. This is not possible.
So, Ali is also a Liar.

Similarly, if Ben is the Truth-teller and we let Ali and Don be the two Liars he is referring to, then Cheryl is the Truth-teller. This is only possible if Ben himself is a Liar.
So, Ali, Ben and Don are the Liars and Cheryl is the only Truth-teller.

(A) Don is the only Liar.
(B) Ben is the only Liar.
(C) Ali and Don are the only Liars.
(D) Ben is the only Truth-teller.
(E) Cheryl is the only Truth-teller.

Section B

Questions 11 and 12 carry 6 marks each.

11. In the following cryptarithm, each letter represents a different digit from 1 to 9.

	A	B
	C	D
	E	F
+		G
1	0	0

If AB is the largest possible 2-digit number that can be formed, what number does AB represent? [Cryptarithm]

Since we want AB to be as large as possible, we make CD and EF as small as possible.
So, we let C = 1 and E = 2.

	A	B
	1	D
	2	F
+		G
1	0	0

We are left with the digits 3, 4, 5, 6, 7, 8 and 9.
B + D + F + G gives the digit 0 in the ones place.
It is not possible to get a sum of 10 from 4 digits in 3 to 9.
The possible sum is 20.
(Note that we can also get a possible sum of 30 but this will make AB smaller.)

Since B + D + F + G = 20, this means A = 5.
Out of the remaining digits 3, 4, 6, 7, 8 and 9, we choose the 4 digits that add up to 20.
3 + 4 + 6 + 7 = 20
So, we let B = 7.

	2	
	5	7
	1	6
	2	4
+		3
1	0	0

AB represents the number 57.

12. Jane writes down 20 different whole numbers. The sum of all these numbers is an even number.

If any 8 numbers are picked from the numbers, the product of all these 8 numbers is an even number.

Find the smallest possible sum of the 20 numbers. [Number Theory]

Any number when multiplied by an even number gives an even number.

No matter how we pick 8 numbers from Jane's numbers, we will definitely pick at least 1 even number.
This is possible only if Jane writes down at least 13 even numbers and the remaining numbers are odd numbers.

Let's say Jane writes down 13 even numbers and 7 odd numbers.
But we are told that the sum of all these numbers is an even number.
This is only possible if there are even number of odd numbers.
So, Jane writes down 14 even numbers and 6 odd numbers.

We are asked to find the smallest possible sum.
So, we write the smallest possible consecutive even and odd numbers.

The 6 odd numbers are 1, 3, 5, 7, 9 and 11.
The 14 even numbers are 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26 and 28.

Using the rainbow method,
$1 + 3 + 5 + 7 + 9 + 11 = 3 \times 12$
$= 36$

$2 + 4 + 6 + \ldots + 24 + 26 + 28 = 7 \times 30$
$= 210$

$36 + 210 = 246$
The smallest possible sum of the 20 numbers is 246.